



IB WORLD SCHOOL 1309 (ZSO13 Gdańsk)



**Mathematics: applications and interpretation SL
syllabus & course of study**

(based on Mathematics: applications and interpretation
guide, first exams 2021, 5h/week)

A. COURSE AIMS:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics.

B. COURSE OBJECTIVES:

1. **Knowledge and understanding:** Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem solving:** Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.
3. **Communication and interpretation:** Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.
4. **Technology:** Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning:** Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.
6. **Inquiry approaches:** Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

C. COURSE OVERVIEW:

C1. Course Content:

C.1.Syllabus component:

- number and algebra
- functions
- geometry and trigonometry
- statistics and probability
- calculus
- the 'toolkit' and mathematical exploration

C.2.Textbook& reference books

M. Haese, M. Humphries *Mathematics: Applications and Interpretation SL 2*
Mathematics: Core Topics SL

C.3 Other requirements

TI-Nspire CX Calculator

D. COURSE OF STUDY

THEMES/ UNITS	Content- topics		Time provision
	Number and algebra	<ul style="list-style-type: none"> - Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. - Arithmetic sequences and series. - Use of the formulae for the nth term and the sum of the first n terms of the sequence. - Use of sigma notation for sums of arithmetic sequences. - Applications - Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life. - Geometric sequences and series. - Use of the formulae for the nth term and the sum of the first n terms of the sequence. - Use of sigma notation for the sums of geometric sequences. - Applications - Financial applications of geometric sequences and series: <ul style="list-style-type: none"> • compound interest • annual depreciation. - Laws of exponents with integer exponents. - Introduction to logarithms with base 10 and e. - Numerical evaluation of logarithms using technology. - Approximation: decimal places, significant figures. - Upper and lower bounds of rounded numbers. - Percentage errors. - Estimation. - Amortization and annuities using technology. - Use technology to solve: <ul style="list-style-type: none"> • Systems of linear equations in up to 3 variables • Polynomial equations 	30 hours

	<p>Functions</p>	<ul style="list-style-type: none"> - Different forms of the equation of a straight line. - Gradient; intercepts. - Lines with gradients m_1 and m_2 - Parallel lines $m_1=m_2$. - Perpendicular lines $m_1m_2 = -1$. - Concept of a function, domain, range and graph. - Function notation, for example $f(x),v(t),C(n)$. - The concept of a function as a mathematical model. - Informal concept that an inverse function reverses or undoes the effect of a function. - Inverse function as a reflection in the line $y=x$, and the notation $f^{-1}(x)$. - The graph of a function; its equation $y = f(x)$. - Creating a sketch from information given or a context, including transferring a graph from screen to paper. - Using technology to graph functions including their sums and differences. - Determine key features of graphs - Finding the point of intersection of two curves or lines using technology. - Modelling with the following functions: <ul style="list-style-type: none"> • Linear models $f(x) = mx + c$. • Quadratic models. $f(x) = ax^2 + bx + c, a \neq 0$; Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y -axis. • Exponential growth and decay models. $f(x) = ka^x + c$, $f(x) = ka^{-x} + c$ (for $a > 0$) $f(x) = ke^{rx} + c$ Equation of a horizontal asymptote. • Direct/inverse variation: $f(x) = ax^n, n \in \mathbb{Z}$ The y-axis as a vertical asymptote when $n < 0$. 	60 hours
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		<ul style="list-style-type: none"> • Cubic models: $f(x) = ax^3 + bx^2 + cx + d.$ • Sinusoidal models: $f(x) = a\sin(bx) + d,$ $f(x) = a\cos(bx) + d.$ <p>- Modelling skills:</p> <ul style="list-style-type: none"> • develop and fit the model, • determine a reasonable domain for a model, • find the parameters of a model • test and reflect upon a model\comment on the appropriateness and reasonableness of a model • use the model – reading, interpreting and making predictions based on the model. 	
	<p>Geometry and trigonometry</p>	<p>- The distance between two points in three-dimensional space, and their midpoint.</p> <p>- Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids.</p> <p>- The size of an angle between two intersecting lines or between a line and a plane.</p> <p>- Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.</p> <p>- The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$</p> <p>- The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$</p> <p>- Area of a triangle as $\frac{1}{2}ab\sin C$</p> <p>- Applications of right and non-right angled trigonometry, including Pythagoras' theorem.</p> <p>- Angles of elevation and depression.</p> <p>- Construction of labelled diagrams from written statements.</p>	60 hours

		<ul style="list-style-type: none"> - The circle: length of an arc; area of a sector. - Equations of perpendicular bisectors. - Voronoi diagrams: sites, vertices, edges, cells. - Addition of a site to an existing Voronoi diagram. - Nearest neighbour interpolation. - Applications of the “toxic waste dump” problem. 	
	<p>Statistics and probability</p>	<ul style="list-style-type: none"> - Concepts of population, sample, random sample, discrete and continuous data. - Reliability of data sources and bias in sampling. - Interpretation of outliers. - Sampling techniques and their effectiveness. - Presentation of data (discrete and continuous): frequency distributions (tables). - Histograms. - Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). - Production and understanding of box and whisker diagrams. - Measures of central tendency (mean, median and mode). - Estimation of mean from grouped data. - Modal class. - Measures of dispersion (interquartile range, standard deviation and variance). - Effect of constant changes on the original data. - Quartiles of discrete data. - Linear correlation of bivariate data. Pearson’s product-moment correlation coefficient, r. - Scatter diagrams; lines of best fit, by eye, passing through the mean point. - Equation of the regression line of y 	60 hours

		<p>on x.</p> <ul style="list-style-type: none"> - Use of the equation of the regression line for prediction purposes. - Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$. - Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. - The probability of an event A is $P(A) = \frac{n(A)}{n(U)}$. - The complementary events A and A' (not A). - Expected number of occurrences. - Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. - Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. - Mutually exclusive events: $P(A \cap B) = 0$. - Conditional probability: $P(A B) = \frac{P(A \cap B)}{P(B)}$ <ul style="list-style-type: none"> - Independent events: $P(A \cap B) = P(A)P(B)$. - Concept of discrete random variables and their probability distributions. - Expected value (mean), $E(X)$ for discrete data. - Applications. - Binomial distribution. - Mean and variance of the binomial distribution. - The normal distribution and curve. - Properties of the normal distribution. - Diagrammatic representation. - Normal probability calculations. - Inverse normal calculations. - Spearman's rank correlation coefficient, r_s. - Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and 	
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	<p>Spearman's rank correlation coefficient, and the effect of outliers on each.</p> <ul style="list-style-type: none"> - Formulation of null and alternative hypotheses, H_0 and H_1. <p>Significance levels. p -values.</p> <ul style="list-style-type: none"> - Expected and observed frequencies. - The χ^2 test for independence: contingency tables, degrees of freedom, critical value. <p>The χ^2 goodness of fit test.</p> <ul style="list-style-type: none"> - The t -test. - Use of the p -value to compare the means of two populations. - Using one-tailed and two-tailed tests. 	
	<p>Calculus</p> <ul style="list-style-type: none"> - Introduction to the concept of a limit. - Derivative interpreted as gradient function and as rate of change. - Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$. - Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}$, $n \in \mathbb{Z}$ - The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where all exponents are integers. - Tangents and normals at a given point, and their equations. - Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where $n \in \mathbb{Z}$, $n \neq -1$. - Anti-differentiation with a boundary condition to determine the constant term. - Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$. - Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. Local maximum and minimum points. - Optimisation problems in context. - Approximating areas using the trapezoidal rule. 	40 hours

Exploration	Knowledge and understanding Problem solving Communication and interpretation Technology Reasoning Inquiry approaches	25 hours
Exam skills	Solving problems from specimen/past papers and exam-style questions Writing mock exams Use of technology	25 hours
Knowledge (revised and/or constructed)	Knowledge relating to the 5 prescribed themes Use of mathematical language and notation Exam strategies and techniques EE ideas, TOK and CAS links explored throughout all the course themes	
Concepts	Approximation Change Equivalence Generalization Modelling Patterns Quantity Relationships Representation Space Systems Validity	
Skills (developed and practised)	Language skills: Productive skills: writing, speaking Receptive skills: listening and reading comprehension Interactive skills: speaking ATL (approaches to learning) skills: <u>self-management skills</u> time management—including using time effectively in class, keeping to deadlines, keeping to class schedules, creating study planners, homework planners and sticking to them self-management—including personal goal setting, organization of learning materials, turning up to class with the right materials organizing information logically, structuring information correctly in essays, and reports using different information organizers for different purposes demonstrating persistence and perseverance, practicing focus and concentration, and overcoming distractions <u>social skills:</u> working in groups—including delegating and taking responsibility, adapting to roles, resolving group conflicts, demonstrating teamwork accepting others—including analyzing others' ideas, respecting others' points of view, and using ideas critically personal challenges—including respecting cultural differences, negotiating	

goals and limitations with peers and with teachers, taking responsibility for own actions

communication skills:

Active listening—including non-verbal communication, and listening to and following directions and procedures

literacy—including reading strategies, using and interpreting a range of contentspecific terminology, interpreting meaning through cultural understanding

being informed—including the use of a variety of media, receiving feedback

informing others—including presentation skills using a variety of media, clear speaking, writing for different purposes and giving feedback

research skills:

accessing information—including researching from a variety of sources, transferring and summarizing information

using a range of technologies, identifying primary and secondary sources

selecting and organizing information— including identifying points of view, bias and weaknesses, using primary and secondary sources, making connections between a variety of resources, collecting, recording and verifying data

referencing—including the use of citing, footnotes and referencing of sources, respecting the concept of intellectual property rights

thinking skills:

generating ideas—including the use of brainstorming

planning—including storyboarding and outlining a plan

inquiring—including questioning and challenging information and arguments, developing questions, developing the skills of critical analysis and using the inquiry cycle

applying knowledge and concepts—including logical progression of arguments

identifying problems—including deductive reasoning, evaluating solutions to problems

creating novel solutions—including the combination of critical and creative strategies, considering a problem from multiple perspectives

self-awareness—including seeking out positive criticism, reflecting on areas of perceived limitation

self-evaluation—including the keeping of learning journals

reflecting at different stages in the learning process on learning experiences in order to support personal development through

thinking about meta-cognition (the way we think and learn)

making connections—including using knowledge, understanding and skills across subjects to create products or solutions, applying skills and knowledge in unfamiliar situations

inquiring in different contexts— including changing the context of an inquiry to gain various perspectives.

<p>Attitudes (encouraged and fostered)</p>	<p>Inquirers They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.</p> <p>Knowledgeable They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.</p> <p>Thinkers They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.</p> <p>Communicators They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.</p> <p>Principled They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.</p> <p>Open-minded They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.</p> <p>Caring They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.</p> <p>Risk-takers They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.</p> <p>Balanced They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.</p> <p>Reflective They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.”</p>
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(More detailed content and specific skills, knowledge, concepts build and/or explored in unit planners)

E. ASSESSMENT

E.1 Assessment outline:

Assessment component	Weighting
External assessment (3 hours)	80%
Paper 1 (1 hour 30 minutes) This paper consists of compulsory short-response questions. A GDC is required for this paper, but not every question will necessarily require its use.	40%
Paper 2 (1 hour 30 minutes) This paper consists of compulsory extended-response questions. A GDC is required for this paper, but not every question will necessarily require its use.	40%
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. Individual oral assessment Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics.(20 marks)	20%

E.2 Assessment criteria

External assessment details—SL

General information

Paper 1 and paper 2

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Papers 1 and 2 will contain some questions, or parts of questions, which are common with HL.

Calculators

For both examination papers, students must have access to a graphic display calculator (GDC) at all times. Regulations covering the types of GDC allowed are provided in Diploma Programme *Assessment procedures*.

Formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to download a copy from IBIS or the Programme Resource Centre and to ensure that there are sufficient copies available for all students.

Awarding of marks

Marks are awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1 and paper 2, full marks are not necessarily awarded for a correct answer with no working. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1

Duration: 1 hour 30 minutes

Weighting: 40%

- This paper consists of compulsory short-response questions.
- Questions on this paper will vary in terms of length and level of difficulty.
- A GDC is required for this paper, but not every question will necessarily require its use.
- Individual questions will not be worth the same number of marks. The marks allocated are indicated at the start of each question.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to test students' knowledge and understanding across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Mark allocation

- This paper is worth **80** marks, representing **40%** of the final mark.

Question type

- Questions of varying levels of difficulty are set.
- One or more steps may be needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Paper 2

Duration: 1 hour 30 minutes

Weighting: 40%

- This paper consists of compulsory extended-response questions.
- Questions on this paper will vary in terms of length and level of difficulty.
- A GDC is required for this paper, but not every question will necessarily require its use.
- Individual questions will not be worth the same number of marks. The marks allocated are indicated at the start of each question.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to assess students' knowledge and understanding of the syllabus in depth. The range of syllabus topics tested in this paper may be narrower than that tested in paper 1.

Mark allocation

- This paper is worth **80** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Question type

- Questions require extended responses.
- Individual questions may require knowledge of more than one topic.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is upon sustained reasoning.

General

Markschemes are used to assess students in all papers. The markschemes are specific to each examination.

Internal assessment criteria—SL and HL

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for their mathematics course if they have not submitted an exploration.

Criterion A	Presentation
Criterion B	Mathematical communication
Criterion C	Personal engagement
Criterion D	Reflection
Criterion E	Use of mathematics

Criterion A: Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The “presentation” criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Criterion B: Mathematical communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The “mathematical communication” criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalized.

Criterion C: Personal engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The “personal engagement” criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student’s work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate’s own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer’s intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer’s intentions.

Criterion D: Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The “reflection” criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student’s understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

Criterion E: Use of mathematics—SL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

The “Use of mathematics” SL criterion assesses to what extent students use mathematics that is **relevant** to the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.

A key word in the descriptor is **demonstrated**. The command term demonstrate means “to make clear by reasoning or evidence, illustrating with examples or practical application”. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.